

# 4.5

## Quadratic Relations of the Form $y = a(x - r)(x - s)$

The Dufferin Gate at the west end of the Canadian National Exhibition in Toronto is a parabolic arch. The arch is approximately 20 m high and approximately 22 m wide. In this section, you will model the arch using an equation of the form  $y = a(x - r)(x - s)$ .



### Tools

- grid paper

### Investigate

How does the equation  $y = a(x - r)(x - s)$  relate to its graph?

1. Create a table of values for the equation  $y = 2(x - 5)(x + 3)$ . Use integer values of  $x$  from  $-6$  to  $6$ .
2. Use finite differences to show that this is a quadratic relation.
3. Graph the relation.
4. Identify the  $x$ -intercepts. How do they relate to the equation?
5. Identify the coordinates of the vertex. How could you use the  $x$ -intercepts to find the coordinates of the vertex?
6. **Reflect** Summarize your findings. Describe how to determine the  $x$ -intercepts and vertex of  $y = a(x - r)(x - s)$ .

7. a) Use your method to analyse the graph of

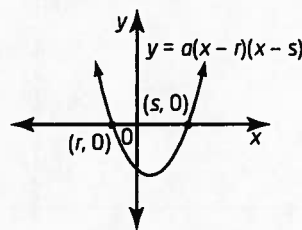
$$y = -\frac{1}{2}(x + 2)(x - 6).$$

- b) Sketch the graph of  $y = -\frac{1}{2}(x + 2)(x - 6)$ .

### Making Connections

You will learn how to convert between the forms  $y = ax^2 + bx + c$  and  $y = a(x - r)(x - s)$  in Chapter 5.

A relation of the form  $y = a(x - r)(x - s)$  is quadratic. The  $x$ -intercepts, or zeros, are  $r$  and  $s$ .





## Making Connections

In Chapter 2, you found the midpoint between two points.

### Example 1 Graph of $y = a(x - r)(x - s)$

Describe the graph of the quadratic relation  $y = 2(x + 1)(x - 7)$ . Sketch the graph and label the x-intercepts, vertex, and axis of symmetry.

#### Solution

Compare  $y = 2(x + 1)(x - 7)$  with  $y = a(x - r)(x - s)$ .

The x-intercepts are  $-1$  and  $7$ .

Due to the symmetric property of a parabola, the x-coordinate of the vertex is on the axis of symmetry, which passes through the midpoint of the line segment connecting the x-intercepts.

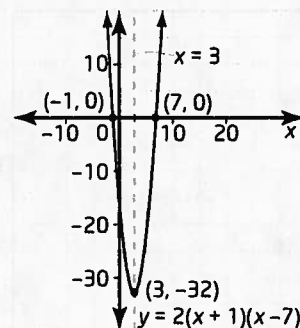
$$\begin{aligned} x &= \frac{-1 + 7}{2} \\ &= 3 \end{aligned}$$

To find the y-coordinate of the vertex, substitute  $x = 3$  into the equation.

$$\begin{aligned} y &= 2(x + 1)(x - 7) \\ &= 2(3 + 1)(3 - 7) \\ &= 2(4)(-4) \\ &= -32 \end{aligned}$$

The vertex is  $(3, -32)$ .

The axis of symmetry has equation  $x = 3$ .



### Example 2 Dufferin Gate

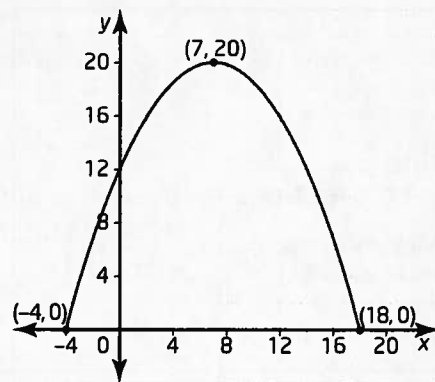
The Dufferin Gate is a parabolic arch that is approximately 20 m tall and approximately 22 m wide.

- Sketch a graph of the arch with the left base located on the x-axis 4 units to the left of the y-axis. Label the x-intercepts and vertex.
- Determine an equation to model the arch.

#### Solution

- The x-intercepts are  $-4$  and  $18$ , so the x-coordinate of the vertex is  $\frac{-4 + 18}{2}$ , or  $7$ .

The maximum height is  $20$  m, so the vertex is  $(7, 20)$ .



- b) Use the x-intercepts to determine that the equation has the form  $y = a(x + 4)(x - 18)$ .

Substitute the coordinates of the vertex and solve for  $a$ .

$$20 = a(7 + 4)(7 - 18)$$

$$20 = a(11)(-11)$$

$$20 = -121a$$

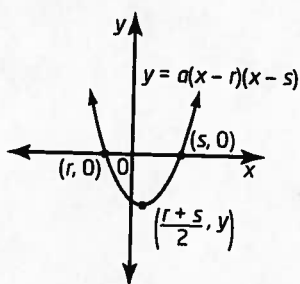
$$a = -\frac{20}{121}$$

An equation representing the shape of the arch is

$y = -\frac{20}{121}(x + 4)(x - 18)$ , where  $y$  is the height, in metres, and  $x$  is the horizontal distance, in metres.

### Key Concepts

- A relation of the form  $y = a(x - r)(x - s)$  is quadratic.
- The x-intercepts, or zeros, are  $r$  and  $s$ .
- The x-coordinate of the vertex is on the axis of symmetry, which passes through the midpoint of the line connecting the x-intercepts.



### Communicate Your Understanding

- C1 With the help of an example, describe a process that you can use to find the vertex of a parabola with equation of the form  $y = a(x - r)(x - s)$ .
- C2 Which is the correct graph of the quadratic relation  $y = 0.5(x - 3)(x + 1)$ ? Explain how you know.

